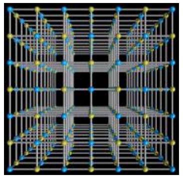
Free Damage Propagation with Memory



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Joint work with

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*Figure : http://www.themolecularuniverse.com/mile/mile1.htm

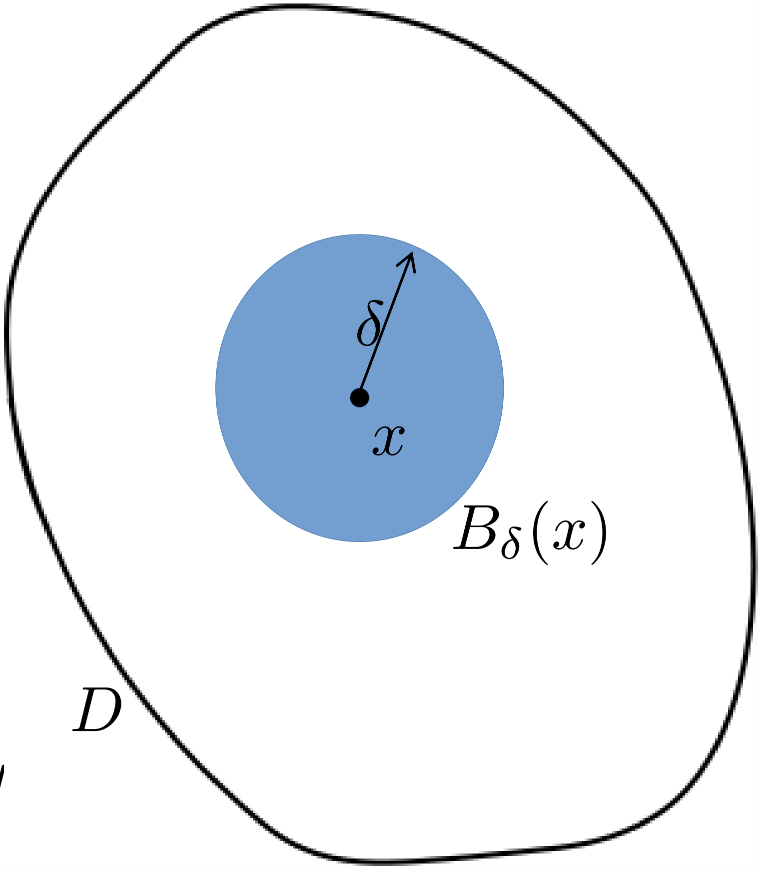
Peridynamic theory

Displacement. Let $u(x)$ denote the displacement of material point $x \in D$

Bond strain. Assuming u is small compared to D, linearized bond strain between material point x, y is given by

$$\mathcal{S} = \mathcal{S}(u) = \frac{u(y) - u(x)}{|y - x|} \cdot e$$

where $e = \frac{y-x}{|y-x|}$.



Hydrostatic strain. $\theta(x)$ at x is given by

$$\theta(x, t; u) = \frac{1}{V_\delta} \int_{D \cap B_\delta(x)} \omega^\delta(|y - x|)|y - x| S(y, x, t; u)\, dy$$

Here $V_\delta = |B_\delta(x)|$, $\omega^\delta(|y - x|)$ is the influence function. We assume $\omega^\delta(|y - x|) = \omega(|y - x|/\delta)$.

Introducing damage to pair interactions

Bond force. Bond force without damage is given by

$$\mathcal{L}^T(u)(x) = \frac{2}{V_\delta} \int_{D \cap B_\delta(x)} \frac{\omega^\delta(|y-x|)}{\delta |y-x|} \partial_S f(\sqrt{|y-x|} S(y,x,t;u)) e_{y-x}\, dy,$$

where f is the bond potential. We assume $f(r) = \alpha r^2/2, f'(r) = \alpha r$. We have

$$\mathcal{L}^T(u)(x) = \frac{2\alpha}{V_\delta} \int_{D \cap B_\delta(x)} \frac{\omega^\delta(|y-x|)}{\delta \sqrt{|y-x|}} S(y,x,t;u) e_{y-x}\, dy.$$

Bond force with damage. We introduc a nonnegative damage function $H^T(u)(y,x,t)$ for each pair of material points y, x. Force due to pairwise interaction is modified to

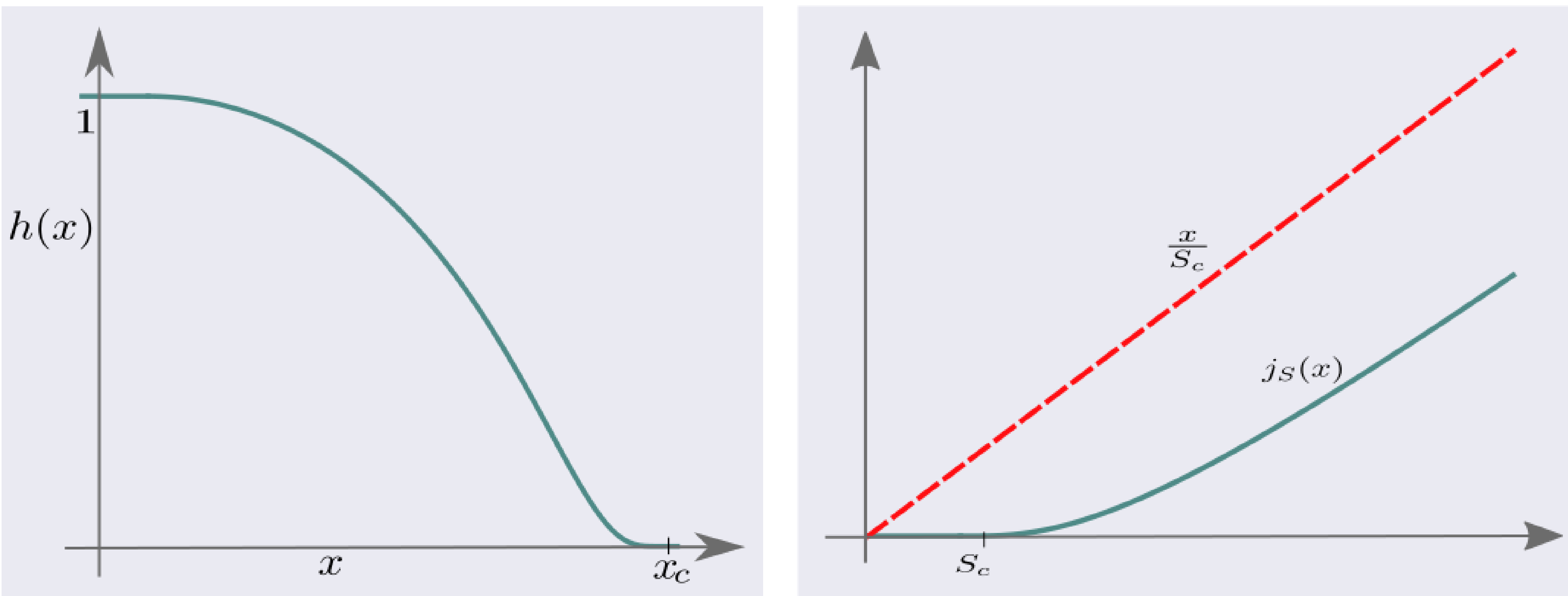
$$\mathcal{L}^T(u)(x) = \frac{2\alpha}{V_\delta} \int_{D \cap B_\delta(x)} \frac{\omega^\delta(|y-x|)}{\delta \sqrt{|y-x|}} H^T(u)(y,x,t) S(y,x,t;u) e_{y-x}\, dy.$$

Damage function

- $H^T(u)(y,x,t) = 1$ if strain $S(y,x,\tau) \leq S_c$ for all $\tau \leq t$.
- $H^T(u)(y,x,t)$ irreversibly decreases when $S(y,x,\tau)$ gets higher than S_c for some $\tau \in [0,t]$.
- Penalty is higher when the deviation of strain $S(y,x,t)$ is higher from the critical strian. This enables us to model two possible scenarios:
- Fatigue in cyclic loading where the strain deviates slightly and only for small amount of time in each cycle. For large loading cycles, the damage accumulated over time can get high.
- Impact loading where the strain's deviation from critical strain is very large and is for very short time.

Damage function

$$H^T(u)(y,x,t) = h\left(\int_0^t j_S(S(y,x,\tau;u))\,d\tau\right)$$



Introducing damage to state interactions

State force. State force without damage is given by

$$\mathcal{L}^D(u)(x,t) = \frac{1}{V_\delta}\int_{D\cap B_\delta(x)} \frac{\omega^\delta(|y-x|)}{\delta^2}\left[\partial_\theta g(\theta(y,t;u)) + \partial_\theta g(\theta(x,t;u))\right] e_{y-x}\, dy,$$

where g is the state potential. We assume $g(r) = \beta r^2/2, g'(r) = \beta r$. We have

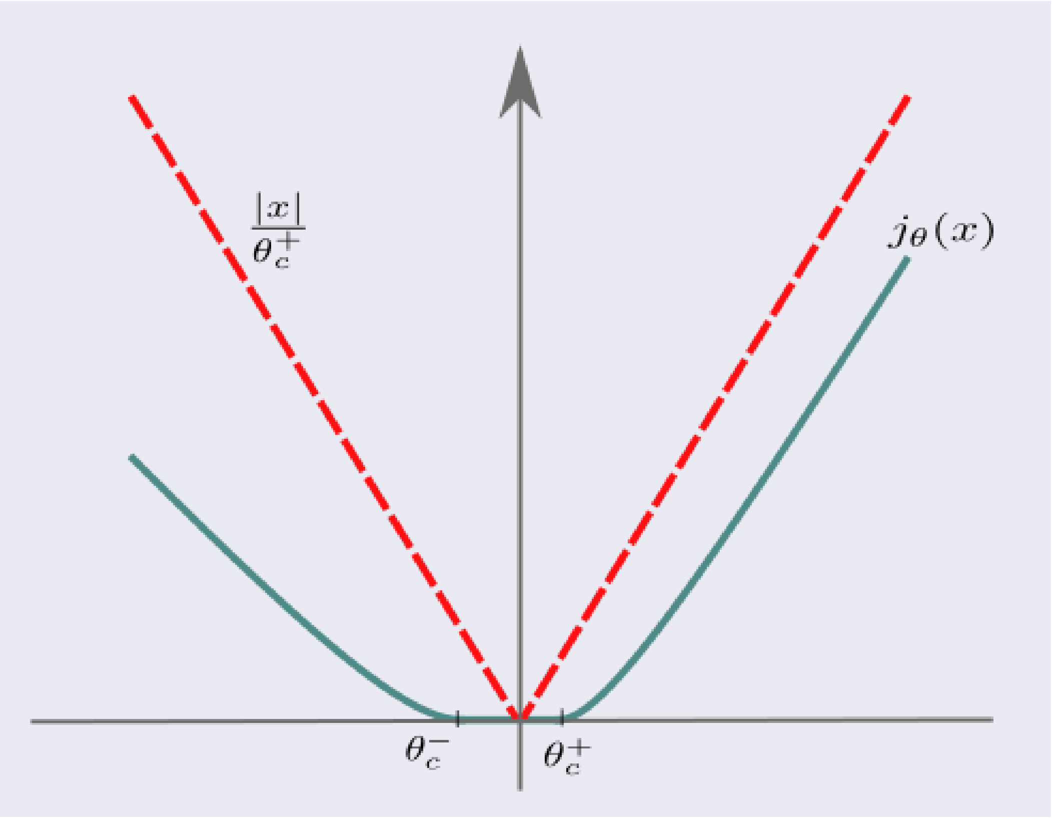
$$\mathcal{L}^D(u)(x,t) = \frac{\beta}{V_\delta}\int_{D\cap B_\delta(x)} \frac{\omega^\delta(|y-x|)}{\delta^2}\left[\theta(y,t;u) + \theta(x,t;u)\right] e_{y-x}\, dy.$$

State force with damage. We introduc a nonnegative damage function $H^D(u)(x,t)$ for each material point x. Force is modified to

$$\mathcal{L}^D(u)(x,t) = \frac{\beta}{V_\delta}\int_{D\cap B_\delta(x)} \frac{\omega^\delta(|y-x|)}{\delta^2}[H^D(u)(y,t)\theta(y,t;u) + H^D(u)(x,t)\theta(x,t;u)]e_{y-x}\, dy.$$

Damage function

$$H^D(u)(x,t) = h\left(\int_0^t j_\theta(\theta(x,\tau;u))\,d\tau\right)$$



Explicit examples of functions used in the model

Function h.

$$h(x) = \begin{cases} \bar{h}(x/x_c), & \text{for } x \in (0, x_c), \\ 1, & \text{for } x \leq 0, \\ 0, & \text{for } x \geq x_c. \end{cases}$$

with $\bar{h} : [0,1] \to \mathbb{R}^+$ is defined as

$$\bar{h}(x) = \exp[1 - \frac{1}{1 - (x)^a}]$$

where $a > 1$ is fixed. Clearly, $\bar{h}(0) = 1$, $\bar{h}(1) = 0$.

Explicit examples of functions used in the model

Function j_S. For a given critical strain $S_c > 0$, we define the threshold function for tensile strain $j_S(x)$ as follows

$$j_S(x) := \begin{cases} \bar{j}(x/S_c), & \forall x \in [S_c, \infty), \\ 0, & \text{otherwise.} \end{cases}$$

where $\bar{j} : [1, \infty) \to \mathbb{R}^+$ is given by

$$\bar{j}(x) = \frac{(x-1)^a}{1+x^b}$$

with $a > 1$ and $b \geq a - 1$ fixed. Note that $j_S(1) = 0$. Here the condition $b \geq a - 1$ insures the existence of a constant $\gamma > 0$ for which

$$j_S(x) \leq \gamma |x|, \qquad \forall x \in \mathbb{R}.$$

Evolution equation and key properties

- **Peridynamic evolution equation.** For a given body force $b(t) : D \to \mathbb{R}^d$, we seek for displacment field u such that

$$\rho \partial_{tt}^2 u(t,x) = \mathcal{L}^T(u)(x,t) + \mathcal{L}^D(u)(x,t) + b(t,x),$$

for all $t \in [0,T]$, with $u(0,x) = u_0(x)$ and $\partial_t u(0,x) = v_0(x)$.

- Peridynamic equation is well posed and solution can be shown to exist in $C^2([0,T]; L^\infty(D;\mathbb{R}^d))$, [1, Theorem 3.1].

- Under sufficient smoothness assumption on displacement and in the absence of damage, the peridynamic operator converges to the linear elastic operator, [1, Theorem 7.1]. Here linear elastic operator has same lamé constant that we showed earlier.

[1] Robert Lipton, Eyad Said, and Prashant K. Jha. Free damage propagation with memory. Journal of Elasticity, 2018.

Related works

- S. A. Silling, *Reformulation of elasticity theory for discontinuities and long-range forces*, Journal of the Mechanics and Physics of Solids, 48 (2000), pp. 175–209.
- S. A. Silling, M. Epton, O. Weckner, J. Xu, and E. Askari, *Peridynamic states and constitutive modeling*, Journal of Elasticity, 88 (2007), pp. 151–184.
- A. Agwai, I. Guven, and E. Madenci, *Predicting crack propagation with peridynamics: a comparative study*, International Journal of Fracture, 171 (2011), pp. 65–78.
- W. Gerstle, N. Sau, and S. Silling, *Peridynamic modeling of concrete structures*, Nuclear Engineering and Design, 237 (2007), pp. 1250–1258.
- E. Emmrich and D. Phust, *A short note on modeling damage in peridynamics*, J. Elast., 123 (2016), pp. 245–252.
- Q. Du, Y. Tao, and X. Tian, *A peridynamic model of fracture mechanics with bond-breaking*, Journal of Elasticity, (2017) DOI 10.1007/s10659-017-9661-2.

Calibration with linear elastic model

- For $d = 2$, the material parameters are related to lamé parameters as follows

$$\mu = \frac{M\alpha}{8}, \qquad \lambda = \mu + \beta M^2, \qquad M = \int_0^1 r^2 \omega(r) dr,$$

where we recall that influence function $\omega^\delta(|y-x|) = \omega(|y-x|/\delta)$, pair potential $f(r) = \alpha r^2/2$, and state potential $g(r) = \beta r^2/2$.

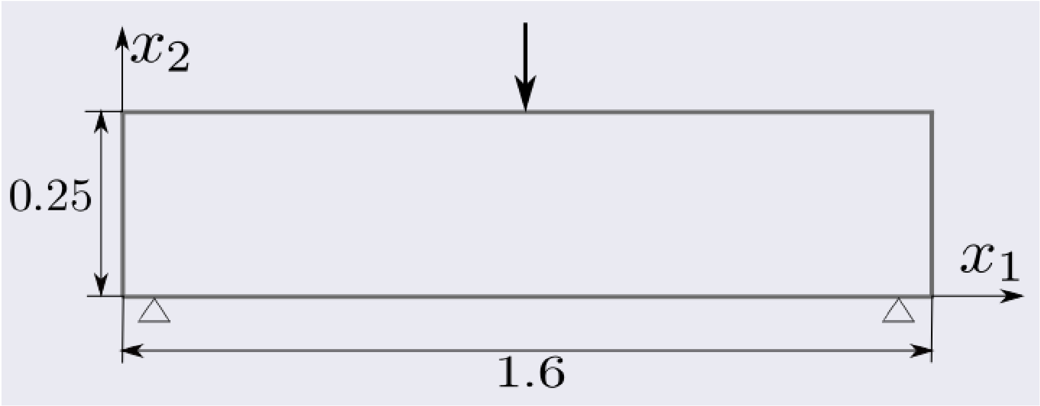
- For $d = 3$, the material parameters are related to lamé parameters as follows

$$\mu = \frac{M\alpha}{10}, \qquad \lambda = \mu + \beta M^2, \qquad M = \int_0^1 r^3 \omega(r) dr.$$

Numerical experiment: Bending damage

- Young's modulus $E = 25$ GPa and Poisson's ratio $\nu = 0.2$.
- Choose $\omega(r) = 1 - r$ to get $\alpha = 540.0$ GPa and $\beta = -270.0$ GPa.
- Bar $D = [0, 1.6\,m] \times [0, 0.25\,m]$, horizon $\delta = 0.1\,m$, mesh size $h = \delta/4$, final time $T = 1.0\,sec$, time step $\Delta t = 2.0 \times 10^{-6}\,sec$.
- **Three point test.** Specified displacement along y-axis at point $P_{load} = (0.8, 0.25)$ is of the form

$$u_2(t) = -t\gamma,\ \gamma = 0.001$$

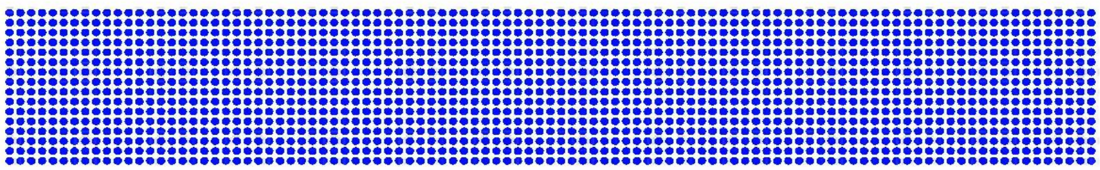


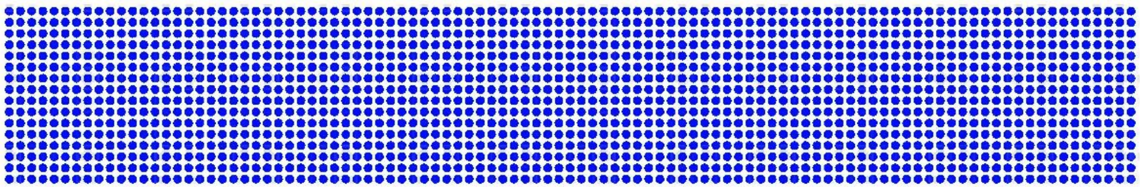
- **Four point test.** Displacement $u_2(t) = -t\gamma$ is specified at point $P_{load,1} = (0.6, 0.25)$ and $P_{load,2} = (1.0, 0.25)$.

Bending damage: Simulation

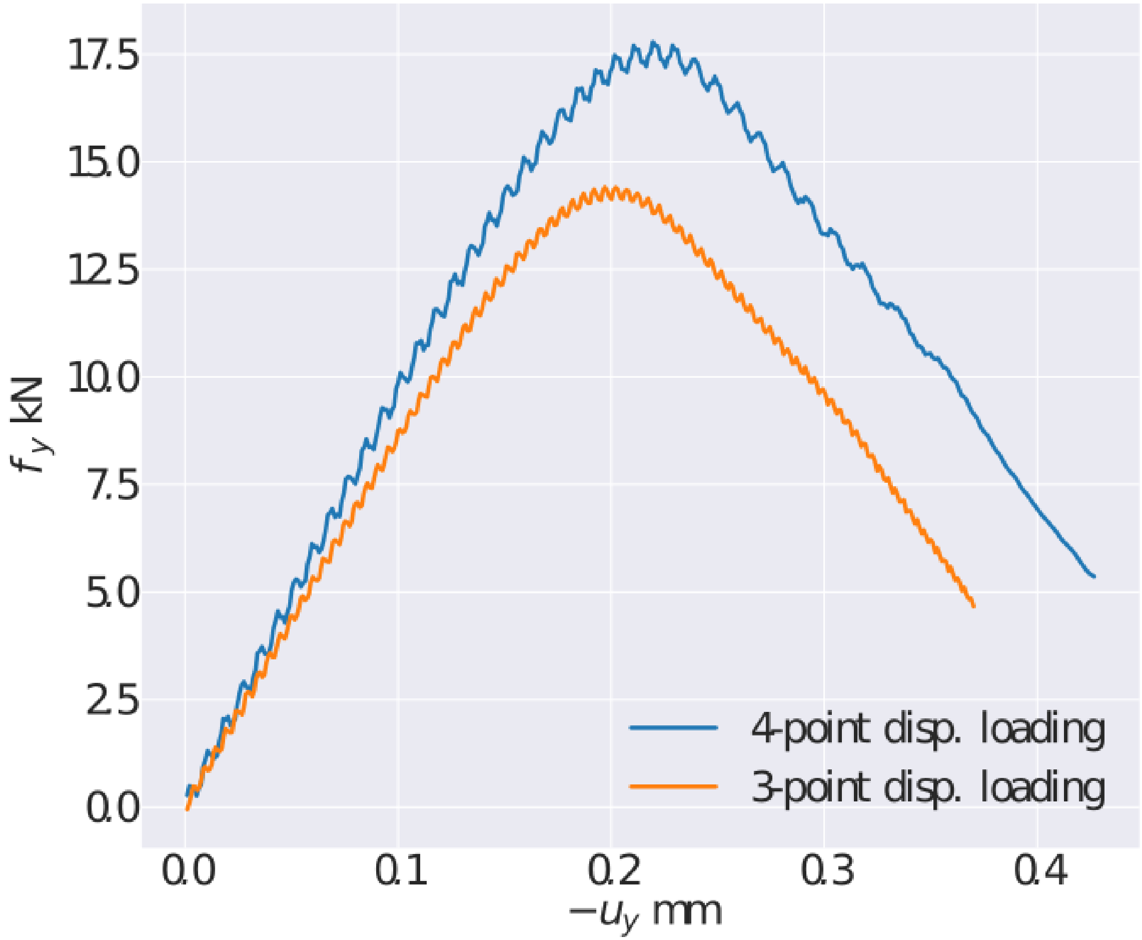
- **Measure of a damage at material point.**

$$\phi(t,x;u) = 1 - \frac{\int_{D\cap B_\delta(x)} H^T(u)(y,x,t)dy}{\int_{D\cap B_\delta(x)} dy}.$$





Bending damage: Deflection vs force curve

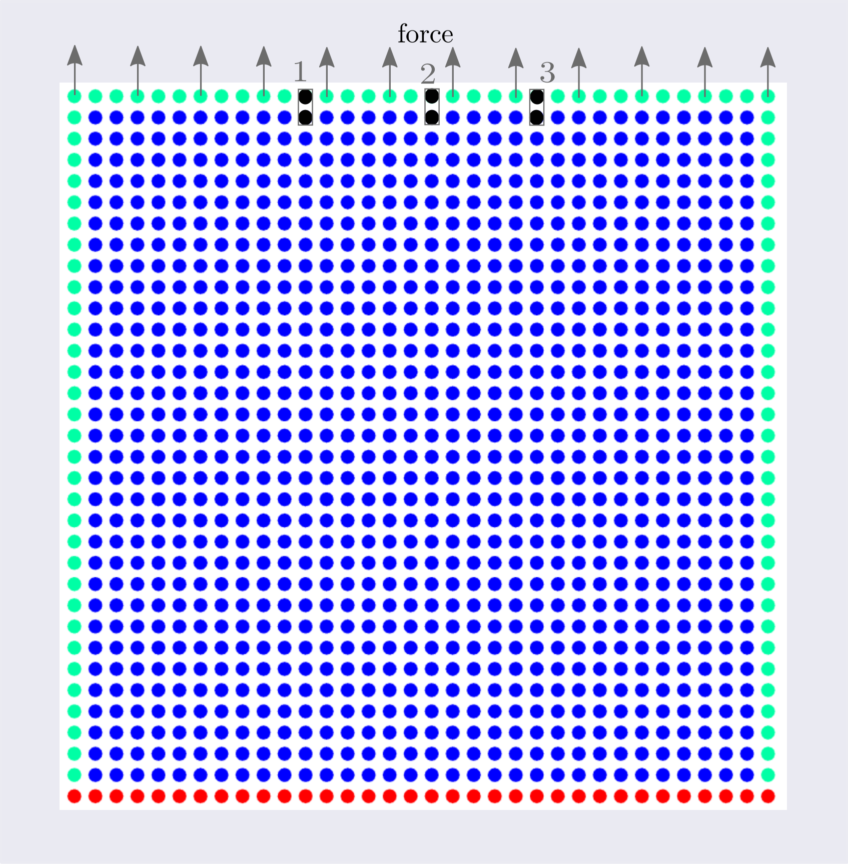


Here we plot the deflection of point at the center $(0.8, 0.125)$ of the beam. The force is the total force experienced by mesh node at P_{load} in case of 3-point test and sum of total force experienced by mesh node $P_{load,1}$ and $P_{load,2}$ in case of 4-point test. We believe the small oscillations seen in the plot are due to the dynamic nature of simulation.

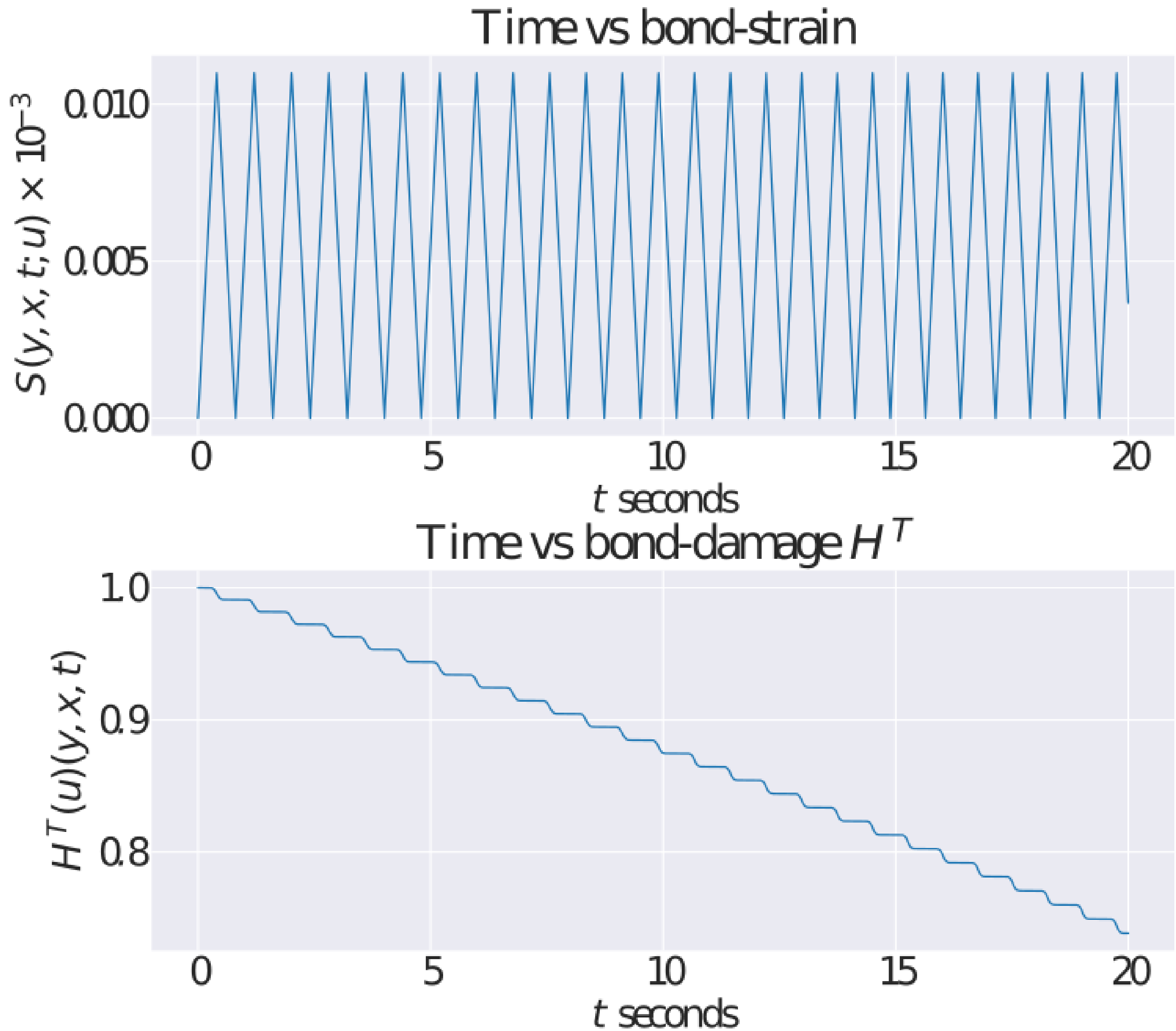
Numerical experiment: Periodic loading

- $D = [0, 1\,m]^2$, horizon $\delta = 0.15\,m$, mesh size $h = \delta/5$, final time $T = 20.0\,sec$, time step $\Delta t = 4.0 \times 10^{-7}\,sec$.

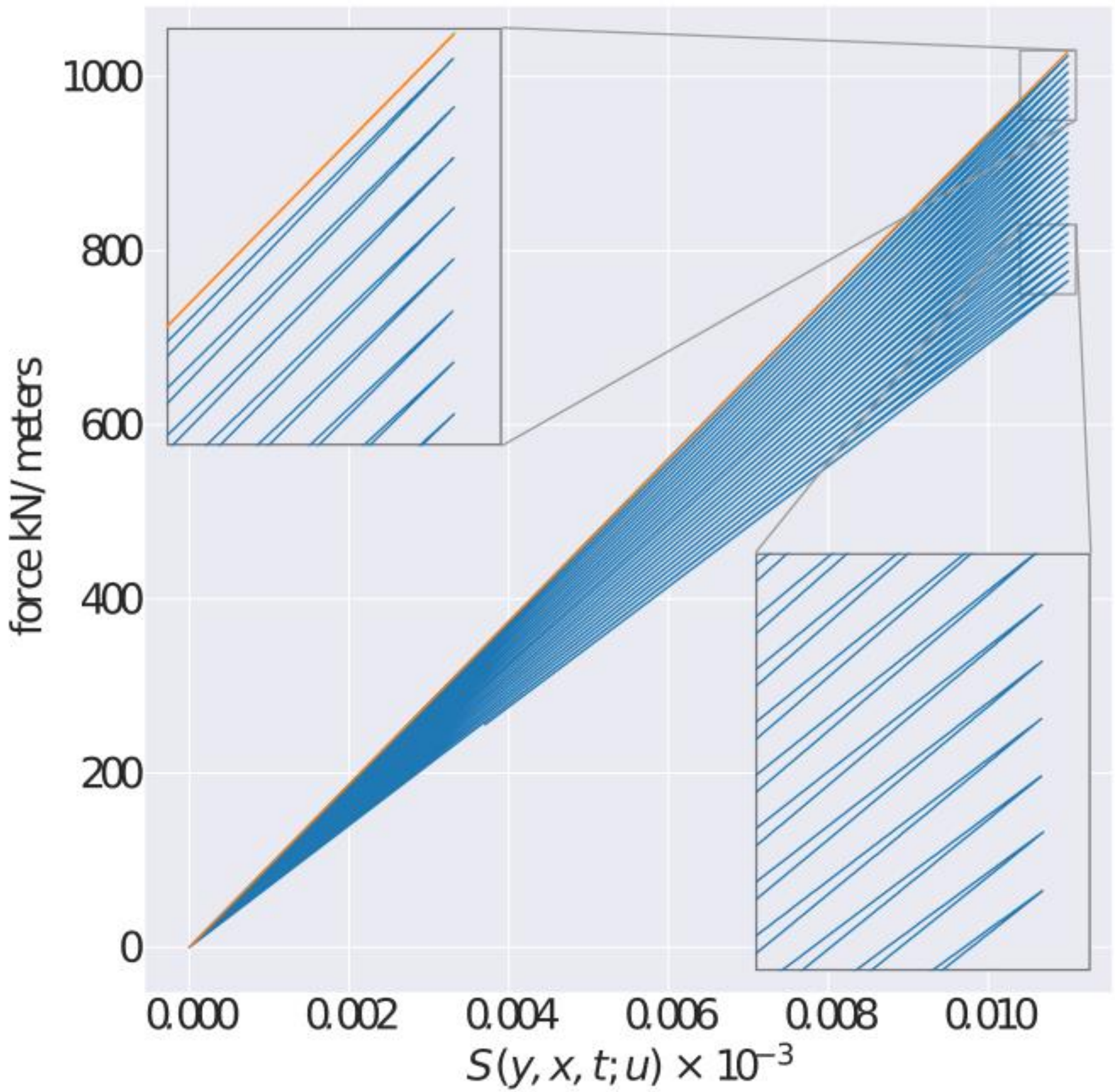
- We apply force on top edge such that maximum strain of bond "2" is $S_{max} = 11.0 \times 10^{-6}$.



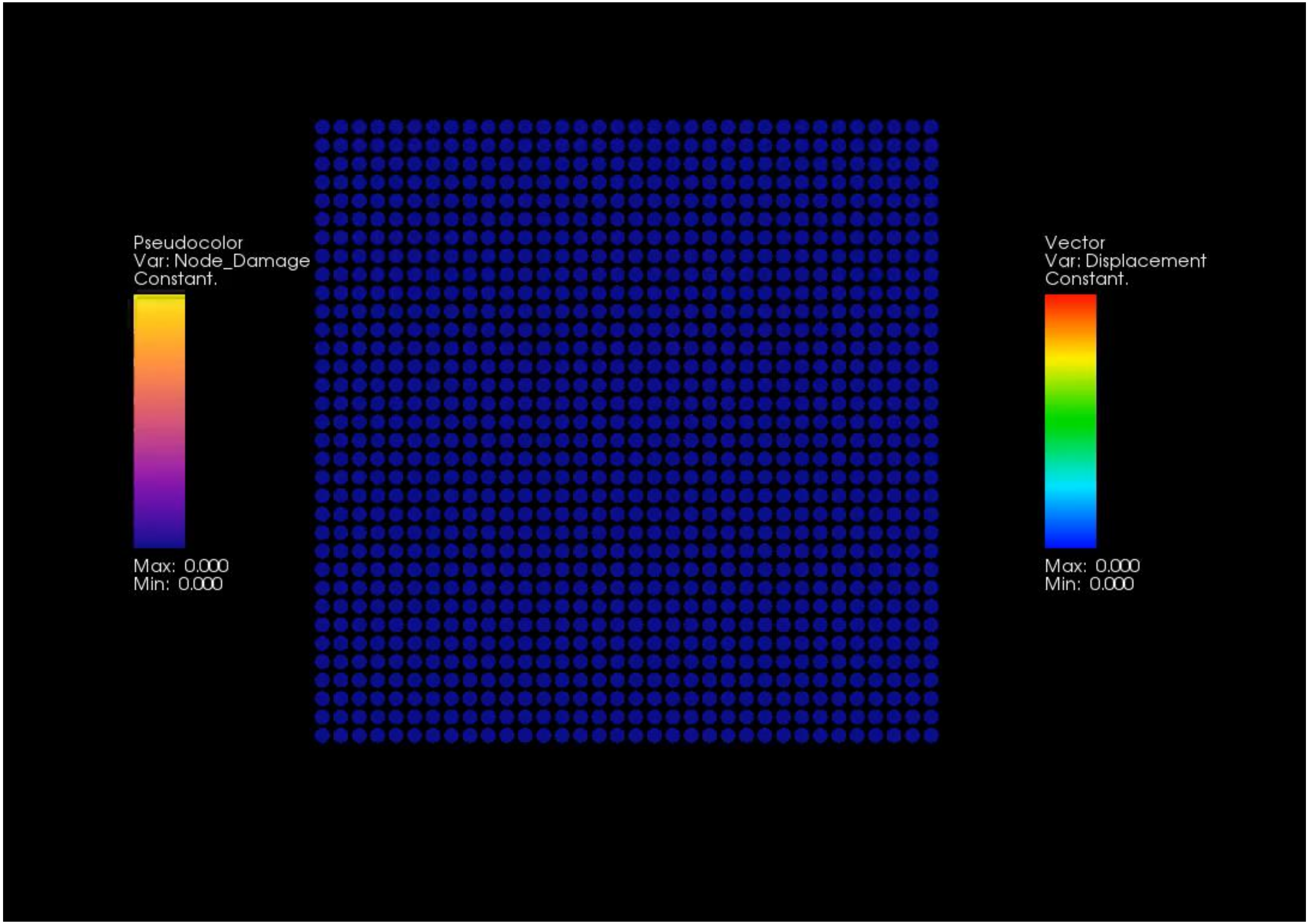
Periodic loading: Time vs strain and damage



Periodic loading: Loss of stiffness with cycles



Numerical experiment: Shear loading

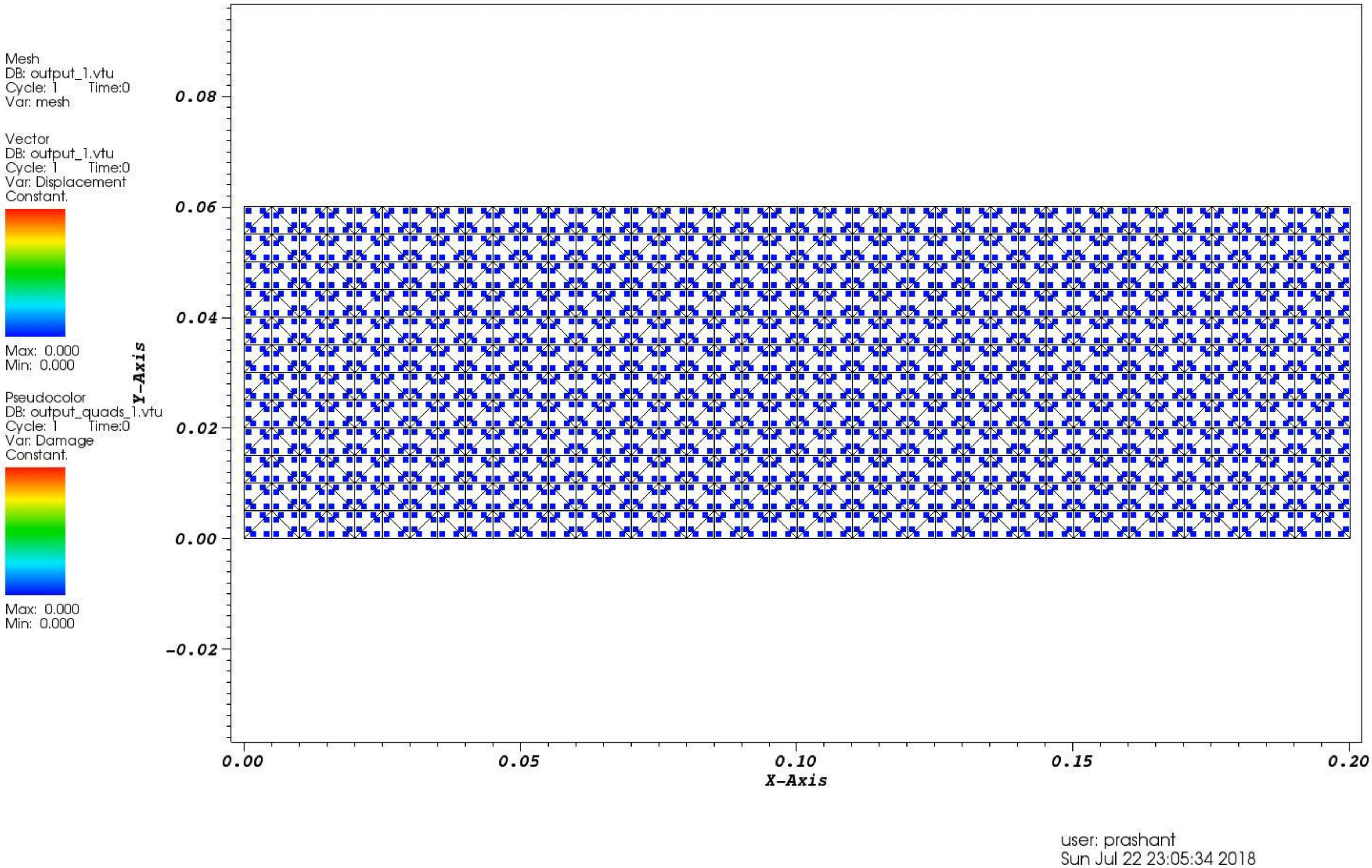


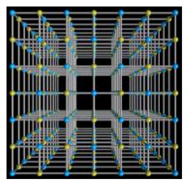
PeridynamicHPX

- Jointly developed with Patrick Diehl
- Soon we will make it open source
- Interested parties can get in touch and we will be happy to share the code
- While other solvers are available, our code is less complex and easy to extend and is good for academic research
- [1] is under review and we are working on second paper where we are implementing Peridynamics using finite element method

[1] P. Diehl, P. K. Jha, H. Kaiser, R. Lipton, M. Levesque. Implementation of Peridynamics utilizing HPX -- the C++ standard library for parallelism and concurrency. Under review, on arxiv.

PeridynamicHPX





Thank You!